GEOPHYSICS, VOL. 45, NO. 12 (DECEMBER 1980); P. 1872–1875, 2 FIGS.

Short Note

The effect of a finite distance between potential electrodes on Schlumberger resistivity measurements—A simple correction graph

E. Mundry*

Resistivity soundings are often carried out using a Schlumberger array. With this configuration, a current I is fed into the ground using two current electrodes A and B placed a distance L apart. At the center of these two electrodes, the voltage U is measured between two potential electrodes M and N placed a distance a apart. The condition $a \ll L$ is necessary to satisfy the assumption of a Schlumberger sounding curve, which is a log-log plot of apparent resistivities

$$\rho_a = k \cdot U/I,\ k = \text{geometric factor}, \qquad (1)$$

versus half the distance between the current electrodes $r = L/2$.

For relatively large distances L, an increase in the distance a between the potential electrodes is generally necessary to obtain a sufficiently large voltage. Thereby, in most cases, the apparent resistivity curve is segmented. The displacement of each segment can be attributed to two effects: (1) the change in the geometry of the configuration, because the ratio a/L can no longer be assumed to be infinitely small (upon expanding the distance a between M and N), and/or (2) inhomogeneities of the resistivities near the potential electrodes. A precise interpretation of the sounding data requires a knowledge of the influence of the first effect, because a change in the geometry of the electrode array may change the form of the curve. The influence of this change decreases as L/a increases. The second effect can be eliminated by a parallel shift of one segment to the other. The construction of correction curves which incorporate the first effect are described below. These curves should provide sufficient accuracy even in field work.

As was shown by Deppermann (1954), the apparent resistivity function $\rho_a^{(a)}(L/2)$ for a symmetrical four-point $AMNB$ configuration (finite a) for a horizontally stratified ground may be obtained by an integration of the apparent resistivities $\rho_a^{(0)}$ for a Schlumberger configuration ($a \to 0$) as follows:

$$\rho_a^{(a)}(L/2) = \frac{\pi}{I} \cdot \frac{(L/2)^2 - (a/2)^2}{a} \cdot \left[V\left(\frac{L-a}{2}\right) - V\left(\frac{L+a}{2}\right) \right], \qquad (2)$$

where V is the potential created at the earth's surface. On the other hand, the resistivity function for an idealized Schlumberger configuration ($a \to 0$) is

$$\rho_a^{(0)}(r) = -\frac{\pi}{I} r^2 \frac{dV}{dr}. \qquad (3)$$

Solving this equation for dV/dr, integrating between $L/2 - a/2$ and $L/2 + a/2$, and inserting the result in equation (2) yields

$$\rho_a^{(a)}(L/2) = \frac{(L/2)^2 - (a/2)^2}{a} \int_{\frac{L-a}{2}}^{\frac{L+a}{2}} \frac{\rho_a^{(0)}(r)}{r^2}\, dr. \qquad (4)$$

Deppermann (1954) used a Lagrange interpolation formula to solve this equation, thus obtaining the $\rho_a^{(a)}$ values in the form of a weighted sum of four

Manuscript received by the Editor November 7, 1979; revised manuscript received April 17, 1980.
*Niedersächsisches Landesamt für Bodenforschung, Postfach 510153, Stilleweg 2, D-3000 Hannover 51, West Germany.

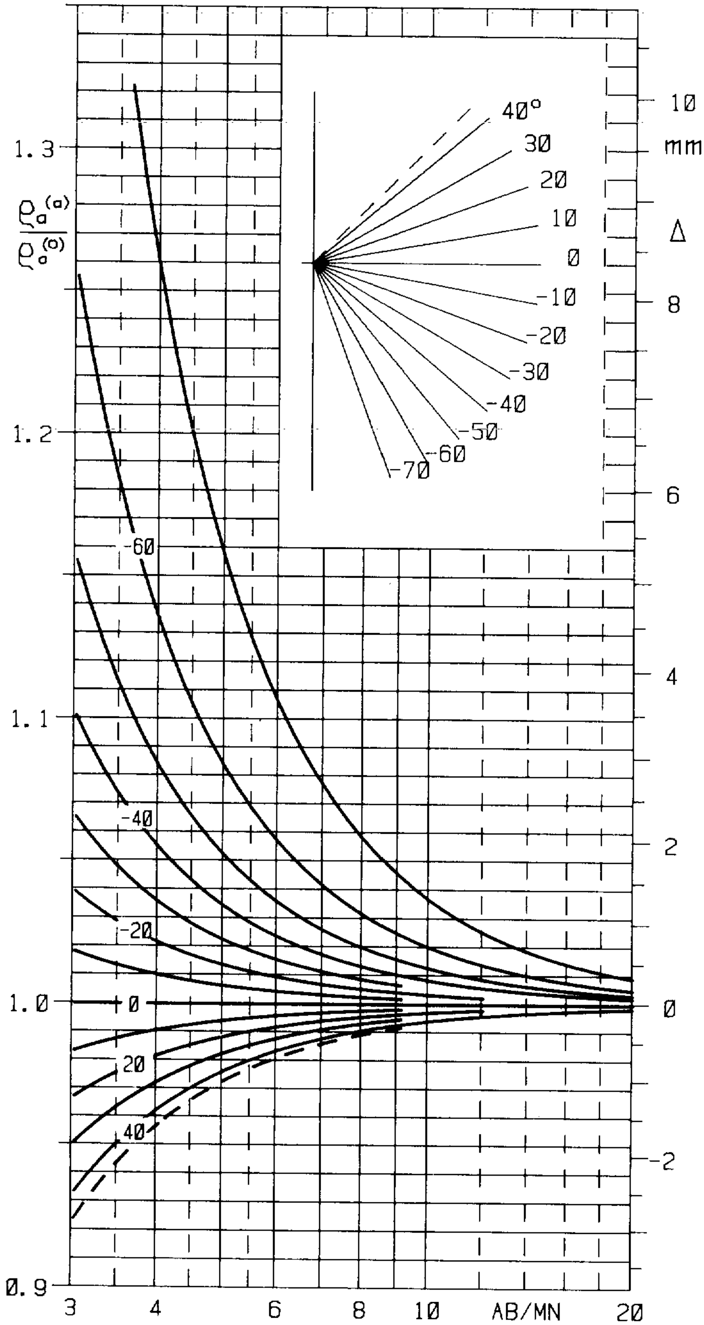

FIG. 1. Ratio of apparent resistivity $\rho_a^{(a)}$ (distance a between potential electrodes M and N) to apparent resistivity $\rho_a^{(0)}$ for a Schlumberger configuration ($a \to 0$) versus the ratio AB/MN for several slopes of the Schlumberger curve.

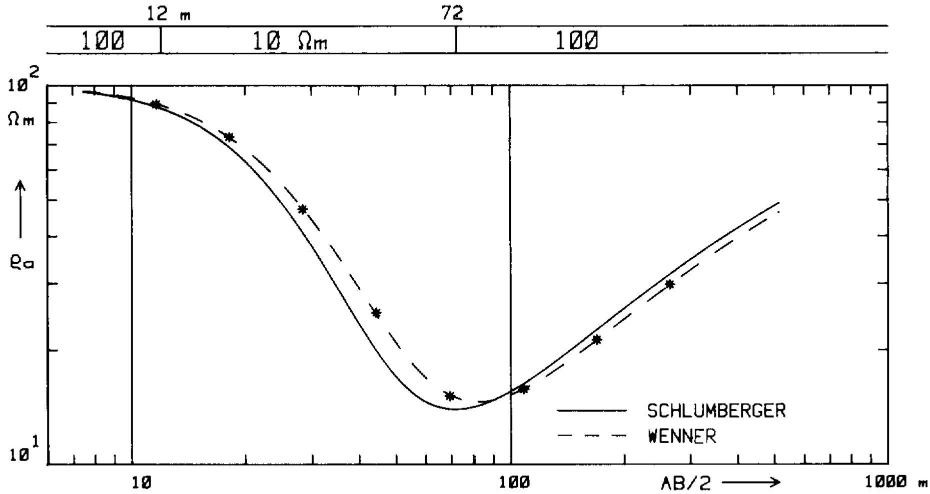

FIG. 2. Schlumberger and Wenner sounding curve for the model in the top of the figure, and Wenner data (*) as obtained with $AB/MN = 3$ from Figure 1.

$\rho_a^{(0)}$ values. A similar procedure can be applied by using linear filter theory (Koefoed, 1979). Here, a simpler integration procedure is used.

To obtain a first approximation for $\rho_a^{(a)}$, the $\rho_a^{(0)}$ curve on a log-log plot is approximated within the interval of integration by a straight line with slope γ:

$$\rho_a^{(0)}(r) \approx \rho_a^{(0)}(L/2)\left(\frac{r}{L/2}\right)^{\gamma}, \qquad L/2 - a/2 \le r \le L/2 + a/2. \tag{5}$$

With this approximation, the integration of equation (4) can be carried out immediately:

$$\rho_a^{(a)}/\rho_a^{(0)} \approx \frac{1-(a/L)^2}{2a/L(\gamma-1)} \cdot \left[\left(1+\frac{a}{L}\right)^{\gamma-1} - \left(1-\frac{a}{L}\right)^{\gamma-1}\right]. \tag{6}$$

This means that, to a first approximation, the ratio of the apparent resistivities is only a function of a/L and γ. For a horizontally stratified ground, the sounding curve can have a maximum slope of 1. By taking the limit $\gamma \rightarrow 1$ in expression (6), one obtains

$$\rho_a^{(a)}/\rho_a^{(0)} \approx \frac{1-(a/L)^2}{2a/L} \cdot \log_e \frac{1+a/L}{1-a/L} \quad (\gamma \rightarrow 1). \tag{6a}$$

Formulas (6) and (6a) were used to construct the correction curves in Figure 1, which show the ratio $\rho_a^{(a)}/\rho_a^{(0)}$ [for various values of the slope (in degrees) of the Schlumberger curve] as a function of the ratio AB/MN (ratio of current electrode spacing to potential electrode spacing). Additionally, the correction Δ (in millimeters) is shown for log paper with a log cycle of 83.33 mm, as is used in Germany. For a quick determination of the slope of the $\rho_a^{(0)}$ curve during field work, a goniometer has been drawn within Figure 1. In practice, the $\rho_a^{(0)}$ curve is taken as that portion of the curve segment that was measured at the largest AB/MN ratio prior to a change of the MN distance.

Since, according to equation (4), the values for $\rho_a^{(0)}$ are weighted with $1/r^2$, the first section of the integration interval has a larger influence on $\rho_a^{(a)}$ than the latter one. Comparison of the $\rho_a^{(a)}$ values obtained from the curves in Figure 1 with exact values shows as a rule of thumb that only the mean slope of the Schlumberger curve within the interval $L/2 - a/2$ to $L/2$ should be used in Figure 1. The exact values were obtained using a computer program based on the following equation for the n-layer case (Kunetz, 1966):

$$\rho_a^{(a)}(L/2) = \rho_1 \left\{ 1 + \frac{(L/2)^2 - (a/2)^2}{a/2} \int_0^\infty K(\lambda) \cdot \left[J_0\left(\lambda\left(\frac{L}{2} - \frac{a}{2}\right)\right) - J_0\left(\lambda\left(\frac{L}{2} + \frac{a}{2}\right)\right)\right] d\lambda \right\} \tag{7}$$

where J_0 is a Bessel function of first kind, zero order, and

$$K(\lambda) = \frac{R_1 \cdot \exp(-2\lambda h_1)}{1 - R_1 \cdot \exp(-2\lambda h_1)}$$

where

$$R_{n-1} = k_{n-1}$$

$$R_i = \frac{k_i + R_{i+1} \cdot \exp(-2\lambda h_{i+1})}{1 + k_i \cdot R_{i+1} \cdot \exp(-2\lambda h_{i+1})}, \quad i = n-2, n-3, \ldots, 1, \tag{7a}$$

where h_i is the thickness of the ith layer, $k_i = (\rho_{i+1} - \rho_i)/(\rho_{i+1} + \rho_i)$ is the reflection coefficient of the ith layer, and ρ_i is the resistivity of the ith layer. As an example, the Schlumberger and the Wenner curves for a specific model are shown in Figure 2. The $\rho_a^{(a)}$ values which were obtained from the Schlumberger curve with the help of Figure 1 and the above-mentioned rule for the $\rho_a^{(0)}$ curve and with $AB/MN = 3$ are also shown (*). The good fit may be observed.

REFERENCES

Deppermann, K., 1954, Die Abhängigkeit des scheinbaren Widerstandes vom Sondenabstand bei der Vierpunkt-Methode: Geophys. Prosp., v. 2, p. 262–273.

Koefoed, O., 1979, Geosounding principles, 1: Resistivity sounding measurements: Amsterdam, Elsevier Scient. Publ. Co.

Kunetz, G., 1966, Principles of direct current resistivity prospecting: Berlin-Nikolassee, Gebrüder-Bornträeger.