Sharp Model Inversion Setup for inversion of geophysical data

- Guidelines and Examples -

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HydroGeophysics Group
Department of Geoscience,
Aarhus University, Denmark

AARHUS UNIVERSITET

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1 INTRODUCTION

1.1 ACRONYMS

Acronym	
AEM	Airborne Electromagnetic
DOI	Depth Of Investigation
ERT	Electrical Resistivity Tomography
GCM	Ground Conductivity Meter
LCI	Laterally Constrained Inversion
SCI	Spatially Constrained Inversion
TEM	Transient Electromagnetic

1.2 INTRODUCTION

Defining a resistivity model of the subsurface to explain a recorded geophysical data set can be done in several ways. 1D resistivity models are typically used to explain or fit airborne EM (AEM) data in an inversion process, but also e.g. electrical resistivity tomography (ERT), and ground conductivity meter (GCM) data are often inverted using 1D resistivity models. The software program Aarhus Workbench process and inverts a number of different geophysical data types, and it offers four main types of model regularization schemes: Layered, smooth (L2), blocky (L1), and sharp.

The L1 and sharp model schemes are the newest implementations in the Aarhus Workbench (via the embedded inversion code AarhusInv) and are both introduced to obtain more pronounced layering (sharper layer boundaries) compared to a smooth L2-model setup, without having to specify an exact numbers of layers as for a few-layer model inversion.

This reported provides a description of the sharp regularization scheme in relation to the layered and smooth L2-model setup, including recommended sharp inversion settings for Aarhus Workbench. Section 3 provides sharp-inversion example for SkyTEM, GCM, and ERT. Similar studies for 1D laterally constrained inversion (LCI) and spatially constrained inversion (SCI) with layered and smooth L2-model setup are reported in references /1/, /2/, and /3/

1.3 RECOMMENDED INVERSION SETTINGS

Recommended sharp-inversion settings for Aarhus Workbench are summarized in the table below. See chapter 4 “Discussion/conclusion“ for comments to the recommended sharp-inversion settings.

Sharp inversion setup

	SCI		LCI	
	Recom-mended	**Working range**	**Recom-mended**	**Working range**
Vertical (intra-layer) constraint, R_V	1.12	1.03-1.2	1.12	1.03-1.2
Lateral (intra-layer) constraint, R_L	1.04	1.01-1.2	1.04	1.01-1.2
Sharpness vertical constraint, S_V	200	5-600	200	5-600
Sharpness lateral constraint, S_L	400	10-1400	200	5-700

Table 1 Recommended sharp inversion settings for Aarhus Workbench

2 INVERSION TYPES

This chapter holds an introduction to the different resistivity model discretization/regularization schemes: Layered, smooth-L2, and sharp. Blocky-L1 is not included here.

Figure 2.1 shows an 1D model inversion using the three different model discretization and regularization schemes of the same synthetic TEM dataset.

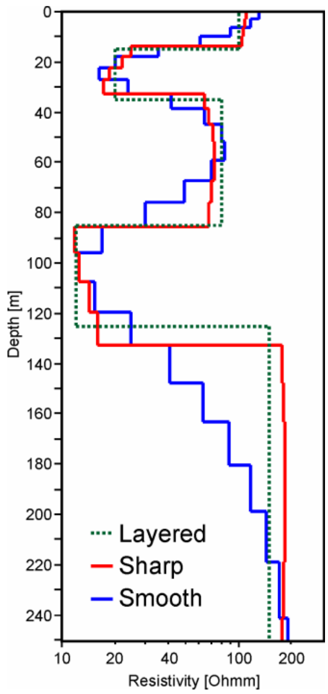

Figure 2.1
1D model inversion with layered, sharp, and Smooth L2-model discretization/ regularization schemes of the same TEM dataset. Note that the layered model is the true model in this case.

2.1 FEW-LAYER MODEL SETUP (PARAMETERIZED INVERSION)

A few-layered resistivity model is characterized by typically 3-5 resistivity layers, where both layer thicknesses and resistivities are model parameters to be determined in the inversion phase. No vertical regularization is applied which results in distinctly layered resistivity models with a fixed number of layers.

In a quasi-layered geological environment, the geophysical layer boundaries can often be correlated directly to distinctive geological layer boundaries, which is useful in the geological interpretation of the geophysical results. Besides the model result, a model parameter analysis is calculated providing estimates of how well the model parameters (resistivities, thicknesses and depths) are determined.

In a SCI or LCI inversion setup the number of layers needs to be uniform for the entire survey. The drawback of this is that layer boundaries in the geophysical resistivity models may not necessarily represent geological boundaries, since the number of layers needed to explain the data might change within the mapping area. Representing strongly sloping geological structures can also be challenging with a layered model setup.

Ground based and airborne TEM data can typically be explained with a layered model holding 3 to 5 layers, ERT data with 4-7 layers, depending on the electrode layout and GCM data with 2-3 layers.

2.2 SMOOTH L2-MODEL SETUP

A smooth L2-model typically consists of 20-30 layers with fixed thicknesses. Consequently, the inversion only needs to determine the layer resistivities. The smooth L2-model setup favors a vertically smooth model output.

The advantages of a smooth L2- model are that the setup is flexible in handling a varying number of geological layers within the survey area and that non-layered complex geological structures, e.g. inclined layer sequences, are easier to detect in a smooth L2 inversion result compared to a layered inversion result.

One disadvantage is that sharp geological layer boundaries appear diffuse and picking geological layer boundaries therefore becomes more subjective as the model example in Figure 2.1 shows. Also, the uncertainty estimates of the model parameters of smooth L2-models will be significantly influenced by the regularization (the vertical and lateral constraints). Consequently, uncertainty estimates may only be used as a relatively indicator of how well the resistivity values are resolved.

2.3 SHARP MODEL SETUP

A sharp model uses the same model discretization as the smooth L2-model, but the model regularization scheme (how the constraints are interpreted in the minimization) is different. A way to look at it is that the sharp model regularization scheme penalizes the *number of resistivity changes* of a certain size, instead of the *absolute resistivity changes* as in the smooth L2-model regularization scheme. Figure 2.2 shows the penalty function imposed in the regularization. The smooth L2-regularization is the green line where the penalty continuously increases with the model variation, i.e. the larger the variation the larger the penalty. The sharp penalty function (take e.g. the red line with $\varepsilon^2 =1$) follows the smooth penalty function for small variations, but then flattens and any larger variation practically has the same penalty.

The sharp model regularization scheme therefore results in a model with few but relatively sharp resistivity transitions, as seen in Figure

2.1. Small variations within the *constant* units are allowed as suggested by the penalty functions in Figure 2.2.

As seen in Figure 2.1, the sharp inversion scheme combines some of the otherwise complementary advantages of the few-layer inversion and the smooth inversion. The sharp inversion scheme allows for relative abrupt changes in resistivities, like a few-layered inversion, while using the fixed layer thicknesses of the smooth model and thereby keeping the objectivity in deciding the number of layers needed to explain the data.

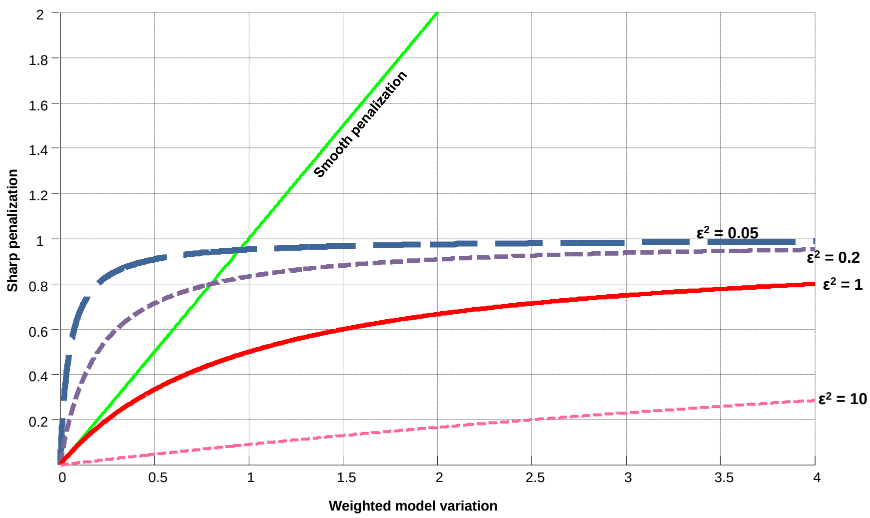

Figure 2.2
Sharp regularization vs smooth L2-regularization. The green line shows the penalty function when doing smooth L2-regularization and the other lines show the sharp penalty function with a varying epsilon handle.

Constraint types
The sharp inversion scheme is also applied for the lateral constraints in a LCI/SCI inversion setups, resulting in lateral sharpness in the same manner as vertical sharpness (see a illustrative example in Figure 3.3).

The sharp inversion scheme works with two set of constraints and regularization parameters: *Intra-layer* constraints (R_L and R_V) and *sharpness* regularization (S_L and S_V).

The intra-layer constraints control the resistivity variations allowed within a *constant* resistivity block, and relates to the first steep part of the penalty function in Figure 2.2. An example is the vertical resistivity variations in the "third" layer (depth 35m – 85m) of the sharp model in Figure 2.1. A very tight intra-layer constraint (small value) results in very homogeneous resistivity blocks and the resulting model will have the same appearance as a few-layer model. In the other end, very lose intra-layer constraints (large value) allows for large intra-layer resistivity variations resistivity block, which effectively breaks the intention of the sharp inversion scheme. An intra-layer constraint between these two extremes is usually preferable, and results in model sections with relative homogenous major resistivity blocks/layers with some minor internal resistivity variations like most of the examples in chapter 3, but exemplified nicely in Figure 3.2. The intra-layer constraint is divided into a lateral (R_L) and a vertical number (R_V), which allows for individual tuning of lateral changes and vertical changes.

The *sharpness* regularization or constraints influence the number of major resistivity transitions. A tight *sharpness* constraint (small number) favors a model output with very few major resistivity transitions and vice versa. The sharpness constraint is similar to the intra layer constraint divided into a lateral (S_L) and a vertical number (S_L), which allows for individual tuning of lateral changes and vertical changes.

The resulting number of major resistivity block in the output model is a trade-off between the intra-layer constraints, the sharpness constraints, and the data.

Working with the sharp inversion requires some adaption, but as guidelines the following can be tried when tuning a sharp inversion:

- Change the sharpness constraint (S_L and S_V) in steps of a factor 1.5 - 2.0.
    - E.g. from 200 to 300 or from 200 to 133.

- Change the intra-layer constraint factors (R_L and R_V) in steps of 30-50% effective change
    - E.g. from 1.10 to 1.15 or from 1.10 to 1.07.

  Note: In a layered environment the lateral constraint should be tighter than the vertical constraint, e.g. R_V: 1.12 and R_L: 1.04. If one is changed the other one should follow to keep the ratio R_V/R_L constant (1.12/1.04 → 1.09/1.03)

A few examples for illustration:

- The model looks too *blocky* with many transitions
    - Allow less transitions by lowering the sharpness constraints: S_V=300 to 200; S_L=300 to 200.

- The *layers* of the model have large variations internally and the overall model only shows small transitions between *layers*
  - Tighten up the intra-layer constraint by lowering the allowed change controlled by the constraint factor: R_V=1.12 to 1.08; R_L=1.06 to 1.04.
  - Optionally, adjust at the same time the sharpness constraints to slightly higher numbers.

- The data fit is poor and the model appears too *homogeneous* with few layers. Especially the lateral variations seem to be limited
  - Allow more transitions via the sharpness constraint, changing only the lateral value: S_V=200 to 200; S_L=200 to 300.

When working iteratively with the numbers it is quite easy to get a feel for the impact the different numbers have. The numbers stated in table should be fairly robust and, in most cases, produce pleasing results.

Mathematical and more detailed descriptions of the sharp inversion scheme is available in reference /4/.

3 EXAMPLES

This section shows inversion examples with the sharp, smooth L2 and layered model setup of ground-based TEM, SkyTEM, ERT, and GCM data. The sharp inversions are all performed with the recommend sharp setting in Table 1. In the smooth and layered inversions standard constrains are used. Other inversion settings (e.g. start model) are specified uniform for the three different inversion types. The inversions of the different datasets are all performed in large LCI or SCI setups, and only a single cross-section for each data type is displayed in this example section.

Besides the resistivity models, the data residual and the depth of investigation estimate (DOI, /5/) of the single resistivity models are included in most cross section plots. The data residual values (how well the resistivity model fits the observed data) are normalized with respect to the data STDs, so that a residual of 1 corresponds to a fit just at the error bar.

3.1 TEM - GROUNDBASED

Synthetic ground-based TEM
The first example is a synthetic TEM study using ground-based data. From a known resistivity structure (the true model, Figure 3.1a) 3D ground based TEM forward responses have been calculated. Noise have been added to the synthetic data and the data have then been inverted in a standard 1D-LCI setup with the sharp, smooth L2 and layered model setup (Figure 3.1b-d). More details about the generation of the synthetic data are given in /3/.

The true model of Figure 3.1a mimic a buried valley structure with a top till-layer of varying resistivity. Note that true model is distinctly sharp in its appearance.

As see in
Figure 3.1 all three inversion setups perform quite well, with the expected higher degree of sharpness for in the layered and sharp inversion results.

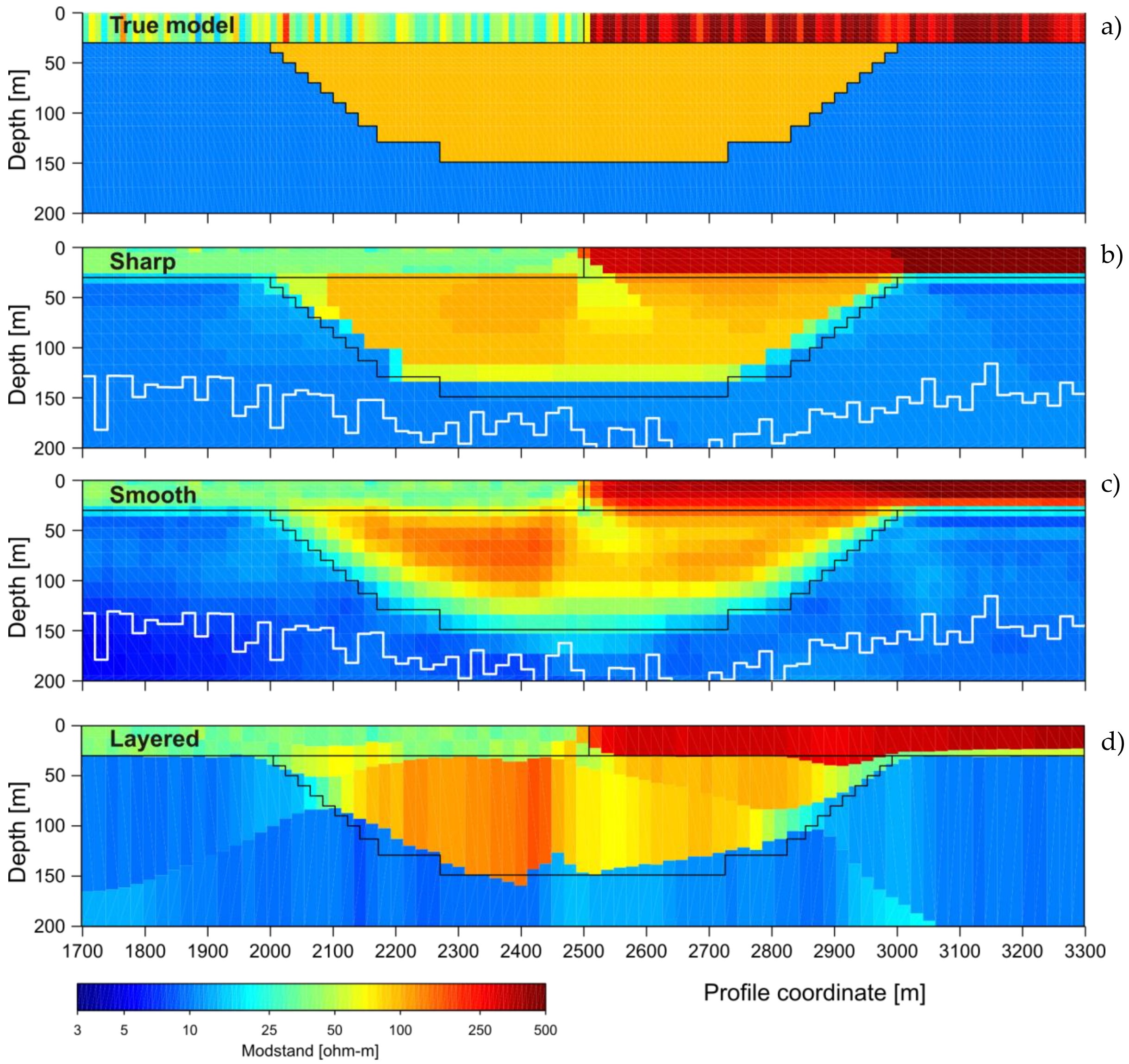

Figure 3.1. Synthetic TEM example. a) Cross section of the in the true 3D-resistivity model, followed by b) sharp, c) smooth L2, and d) 4-layered 1D-LCI inversion result. The black lines outline the main structure of the true model, while the white line marks the DOI estimate of the single models.

SkyTEM

The three 3.2 km long cross-sections in Figure 3.2 are inversion examples of SkyTEM data from the volcanic island of Mayotte. It is clear that the *sharp* model handles the changing numbers of volcanic layers much better that the in the *layered* model section. The layer boundaries in the *smooth L2* sections are as expected smooth. Note that the three model sections fit the data equally good (the black line).

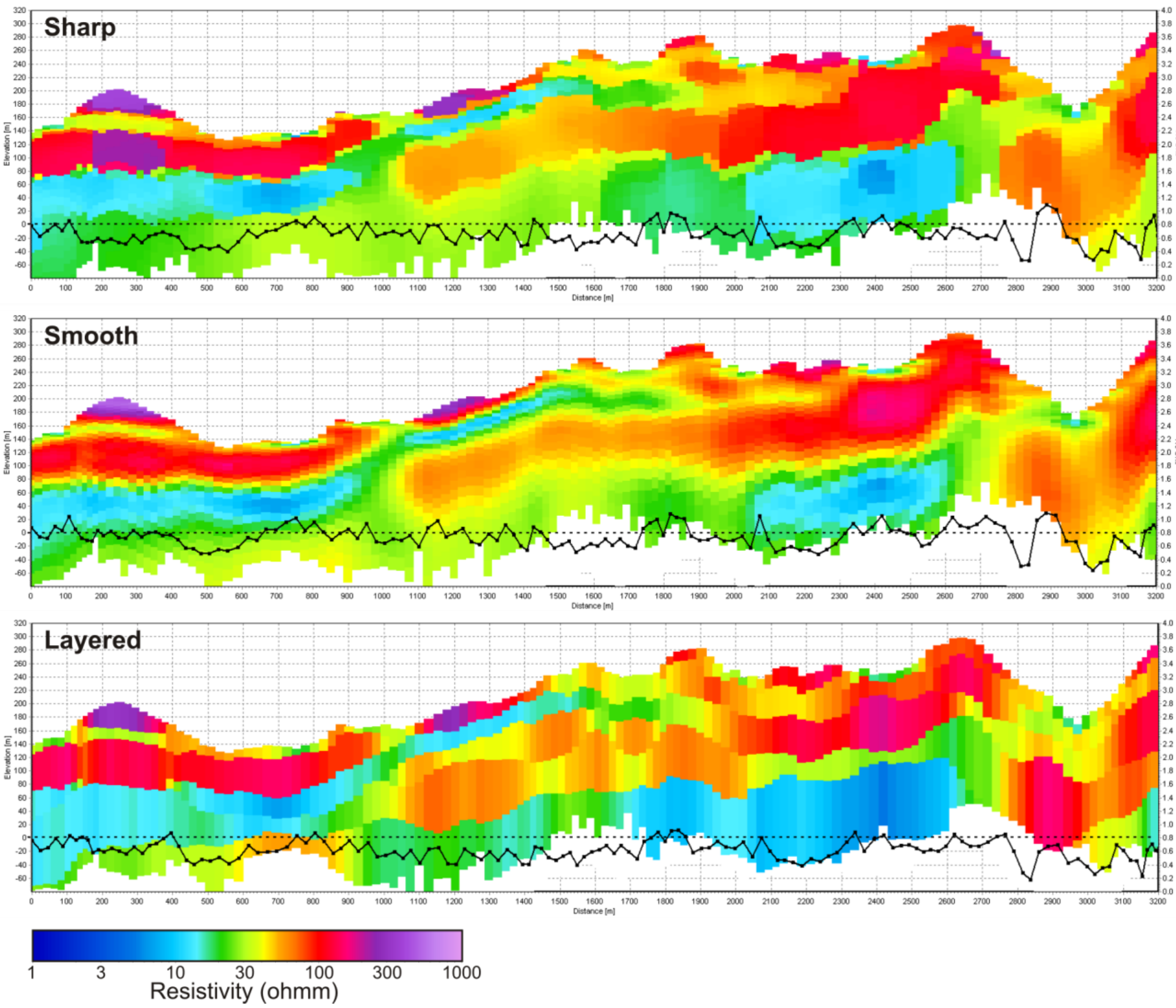

Figure 3.2. SkyTEM cross-section example from the volcanic island of Mayotte, ~3.2 km. The resistivity models are terminated at the estimated depth of investigation. The black curves show the data residuals (the model fit to the data) for the individual soundings. The dashed black line marks a data residual of 0.8 for easy comparison between the sections.

3.2 ELECTRICAL RESISTIVITY TOMOGRAPHY

2D-inversion results of an ERT dataset, are showed in Figure 3.3. The ERT data set was recorded using four 100 m cables with a uniform electrode spacing of 5 m. Using the roll-a-long technique a total 700 m data was collected using gradient configurations.

The 2D-sharp and 2D-smooth L2 inversions were carried out with the AarhusInv inversion code, while the last section in Figure 3.3 is a RES2DINV (Geotomo Software) inversion result using a blocky L1-norm solution (in RES2DINV referred to as *robust inversion*) and a *Vertical/Horizontal flatness ratio* of 0.5.

The 2D-sharp model actually fits the data better the 2D smooth L2-model, but both model types fits the data well within the assigned data STD of 3%. The RES2DINV model also fit the data very well, with a RMS value of 1.22. The data fit values from the AarhusInv and Res2Dinv are not directly comparable but are in the same range.

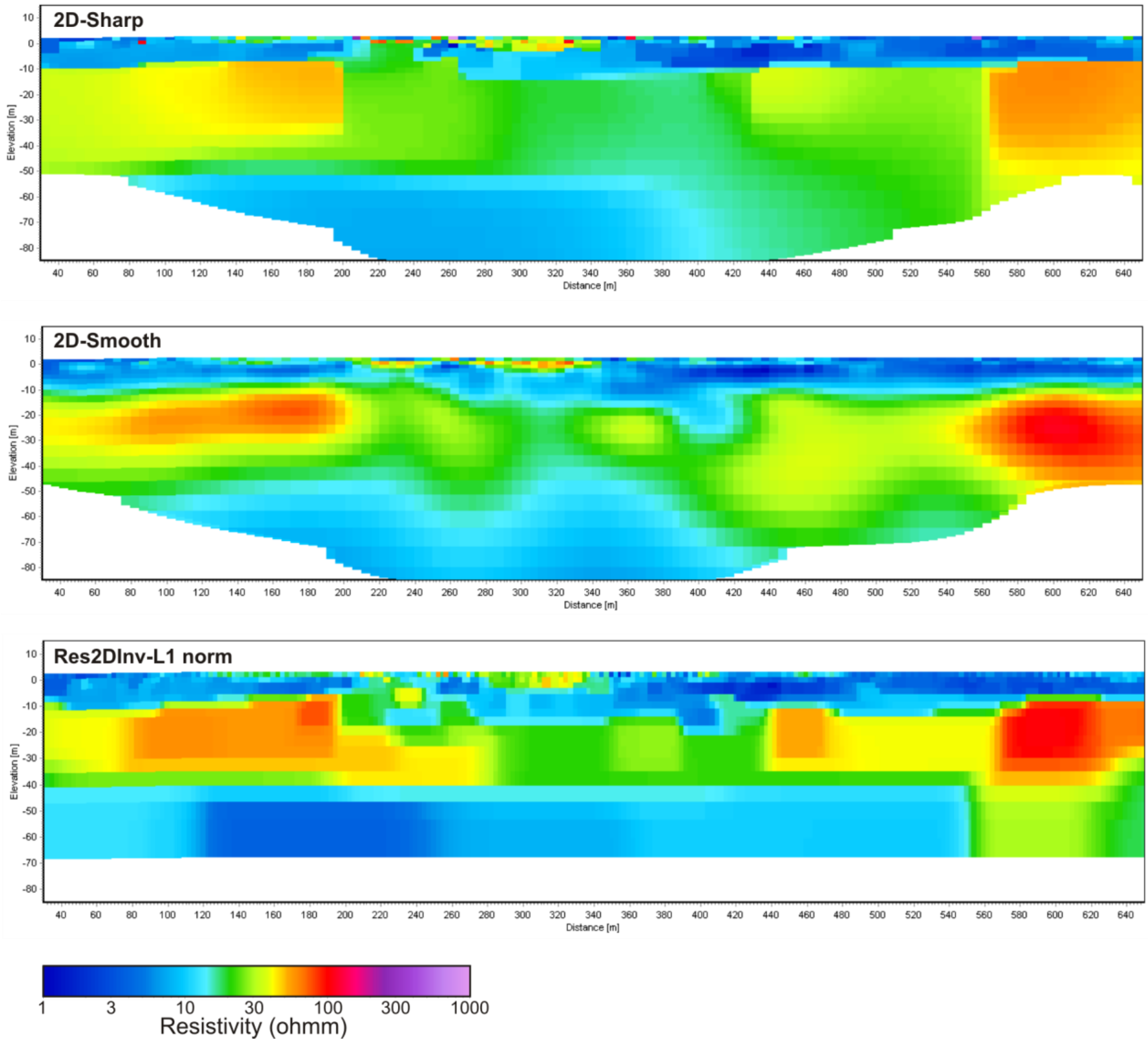

Figure 3.3. 2D-inversion of ERT-data, cross sections example, ~650 m. The sharp and smooth L2-model sections are terminated at the estimated DOI and have a data residual of 0.57 and 0.74 respectively. For the RES2DINV section a L1-norm solution was used, resulting in a RMS data fit of 1.22. The RES2DINV section is discretized to an auto calculated depth by the RES2DINV program and no contouring or smoothing of the model blocks is applied.

3.3 GROUND CONDUCTIVITY METER

The cross-section examples in Figure 3.4 are inversion results of ground conductivity meter (GCM or EMI) data from Jyllinge, Denmark. The data are from a DUALEM-421s instrument. The DUALEM-421s instrument operates with three coil separations (1, 2, 4 m) recording in both horizontal co-planar and perpendicular co-planar configurations, resulting in a total of six configurations or data points per sounding. The DUALEM-421s maps the upper ~5-8 m, with a sounding distance of 1-2 m.

In this case the sharpness seems more pronounced for the sharp GCM example in Figure 3.4 compared to the sharp inversion results of the other data types (TEM, SkyTEM, and ERT). This is likely due to a lower information density in the GCM dataset compared to e.g. SkyTEM, and the inversion result of the GCM data will therefore be stronger influenced by the regularization.

The data residuals for the three inversion results in Figure 3.4 are again very similar. For the layered inversion, a 2-layred model might even be sufficient to explain the data in most parts of this case, but 3 layers were chosen for easier comparison.

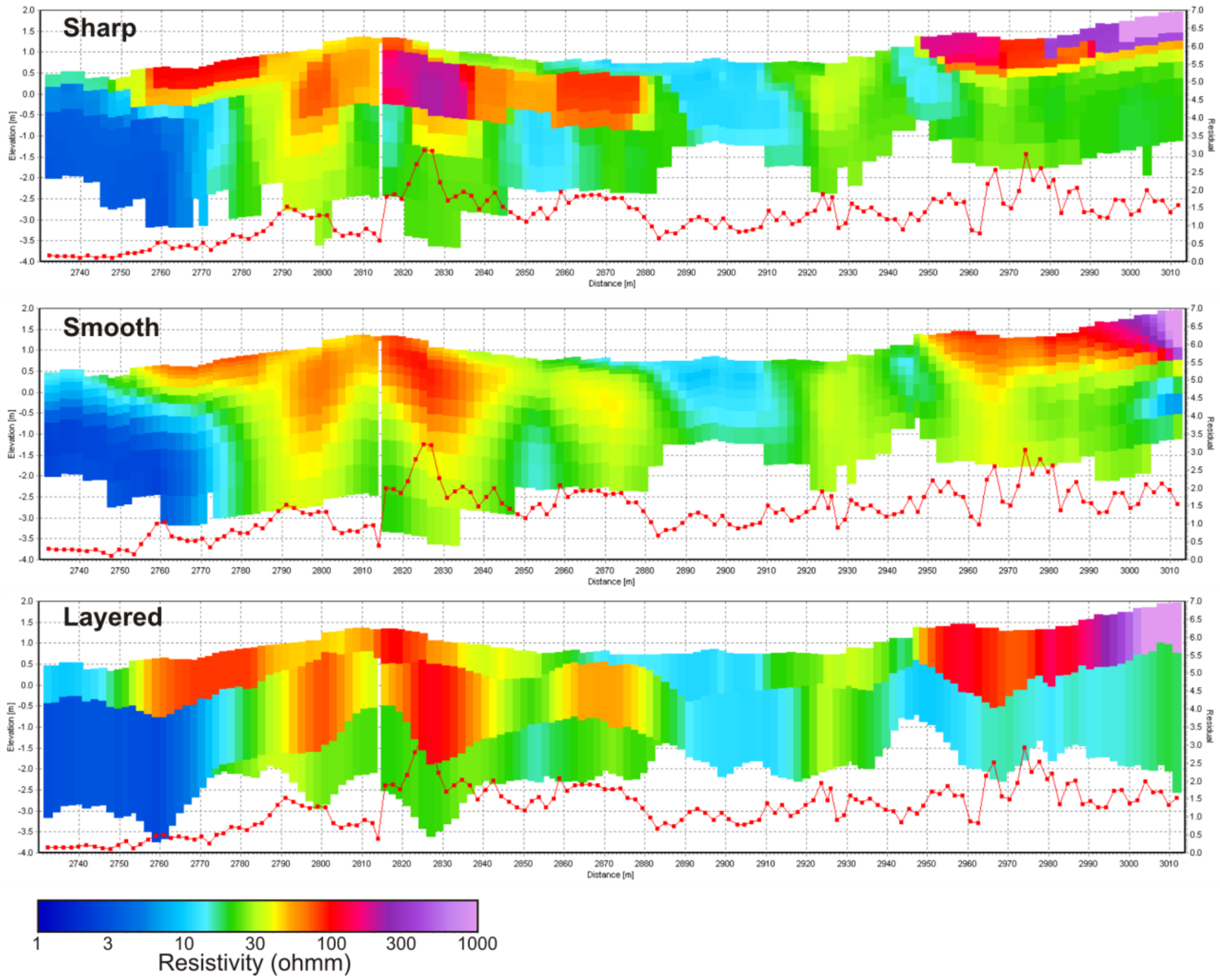

Figure 3.4 GCM cross section example, ~280 m. The resistivity models are terminated at the estimated depth of investigation. The red curves show the data residuals (the model fit to the data) for the individual soundings.

4 DISCUSSION/CONCLUSION

The sharp inversion regularization scheme is a way to obtain blocky or layered model appearances while using a flexible multi-layer model discretization. With these sharp models an interpretation into geological units is supported directly.

Here, a number of default settings are presented together with a range of applicable values and a tuning strategy when optimizing an inversion result.

The sharp procedures presented here are fairly robust, but compared to *standard* smooth L2 model regularization it is lightly more complex and it can require a bit more fine-tuning and hands-on experience to get exactly the desired results. Though, it is our experience that the default values are quite robust with a large range of data types and applications.

The implementation utilizes two main handles; 1) an *intra-layer* constraint which controls the variations within *constant* blocks or layers of resistivity, and 2) a *sharpness regularization* which controls the number of sharp resistivity transitions. Both handles can be controlled individually in the vertical and lateral directions.

Through a number of examples all produced using the default settings this report illustrates the wide applicability of a sharp inversion methodology.

5 REFERENCES

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